Quantum thermalization in disordered spin systems using fluctuation-dissipation relations

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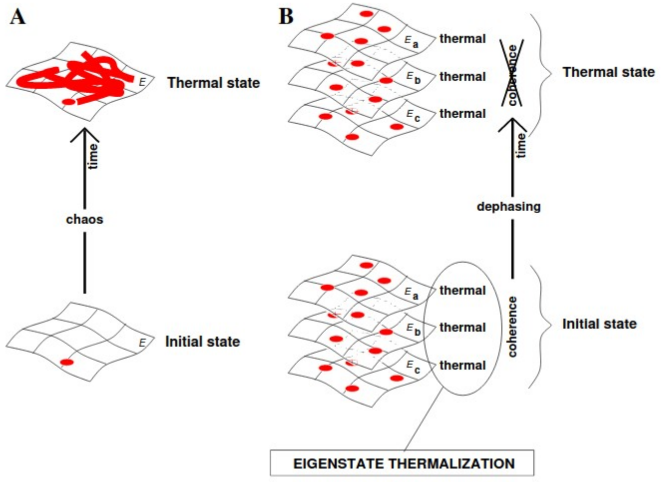


Rigol et al. Nature volume 452, pages 854–858 (2008)

Introduction

Deutsch: Phys. Rev. A 43, 2046

Eigenstate thermalization hypothesis (ETH)

$$O_{mn} \simeq \bar{O}\delta_{mn} + \sqrt{\frac{\overline{O^2}}{\mathcal{D}}} R_{mn}$$

~ every eigenstate looks thermal

Only diagonal part!

Measure late time expectation values

$$\langle \hat{O}(T) \rangle, \quad \langle \hat{O}_1(T)\hat{O}_2(T) \rangle$$

and check for equilibration

Schuckert & Knap: Phys. Rev. Res. 2, 043315 (2020)

Two-time expectation values

$$\langle \hat{O}_1(T_1)\hat{O}_2(T_2) \rangle$$

Use Fluctuation-Dissipation relations to check

→Theory independent!

→necessary condition

ETH ⇒ FDR

Fluctuation-Dissipation Relations (FDR)

Thermal Fluctuations

Statistical function:

$$F(T,\tau) = \frac{1}{2}\left\langle \left\{ \hat{A}(T+\tau), \hat{B}(T-\tau) \right\} \right\rangle$$

Measure via:

- Ancilla + Noise
- Random unitaries *Elben PRL 120, 050406*

Dissipation of Perturbations

Spectral function:

$$\rho(T,\tau) = \left\langle \left[\hat{A}(T+\tau), \hat{B}(T-\tau) \right] \right\rangle$$

Measure via:

- Linear response *Vermersch PRX 9, 021061*
- Ramsey sequences *Knap PRL 111, 147205*

Fluctuation-Dissipation relation

$$F(\omega) = n_\beta(\omega)\rho(\omega)$$

$$n_\beta(\omega) = \frac{1}{2} + \frac{1}{e^{\beta\omega} - 1}$$

⟹ rearrange

$$FDR(\omega) = \log\left(\frac{1}{\frac{F(\omega)}{\rho(\omega)} - \frac{1}{2}} + 1 \right) \stackrel{!}{=} \beta\omega$$

Standard Model for Many-body Localization (MBL)

$$\hat{H} = J\sum_i \hat{\vec{S}}^{(i)}\hat{\vec{S}}^{(i+1)} + \sum h_i \hat{S}_z^{(i)}$$

$$h_i \sim \mathcal{U}(-h, h)$$

Transition @ h=3.5

Thermal phase (ETH)

Eigenstates look locally thermal

MBL phase

Eigenstates localized

Local conserved quantities

Pal & Huse Phys. Rev. B 82, 174411 (2010)

Using exact diagonalization

$$|\psi_0\rangle = |\uparrow\downarrow\ldots\uparrow\downarrow\rangle_x$$

$$\hat{A} = \hat{B} = \hat{S}_z^{(1)}$$

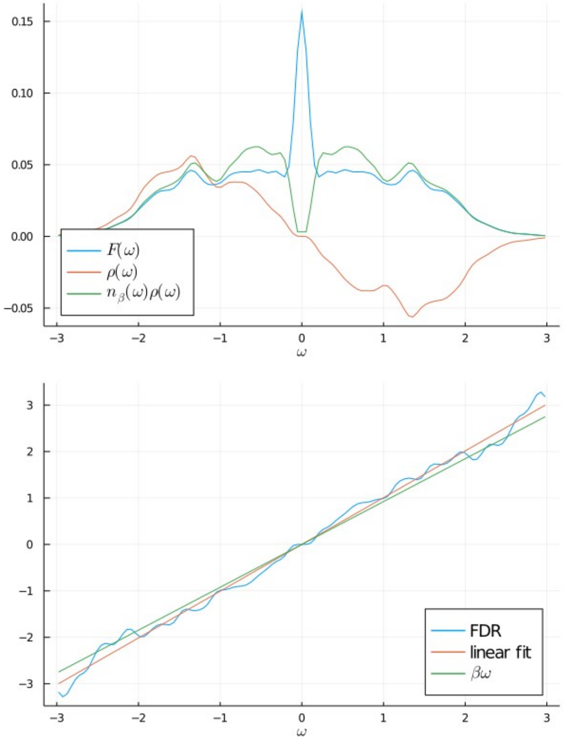
$$T \gg 1$$

Extract inv. Temperature β:

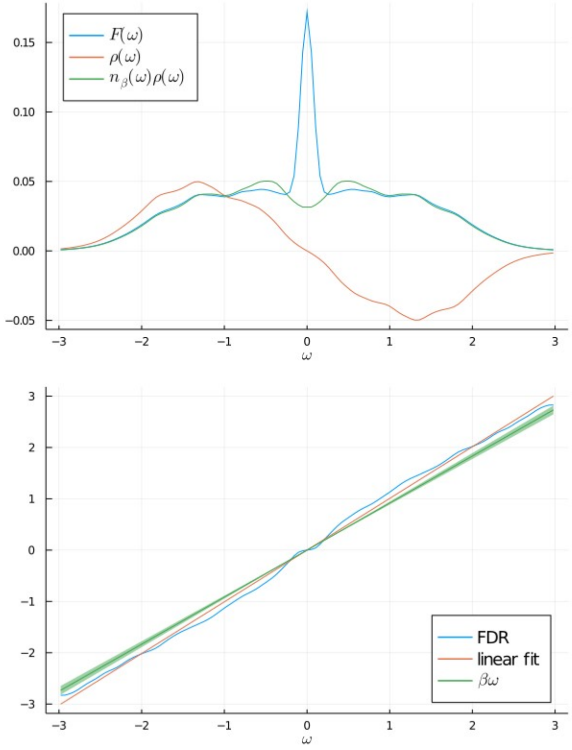
$$\langle\psi_0|\hat{H}|\psi_0\rangle \overset{!}{=} \mathrm{Tr}\hat{H}e^{-\beta\hat{H}}$$

MBL Model (weak disorder – thermalizing phase)

Single shot



Disorder average



$$F(\omega) = n_\beta(\omega)\rho(\omega)$$

Parameters

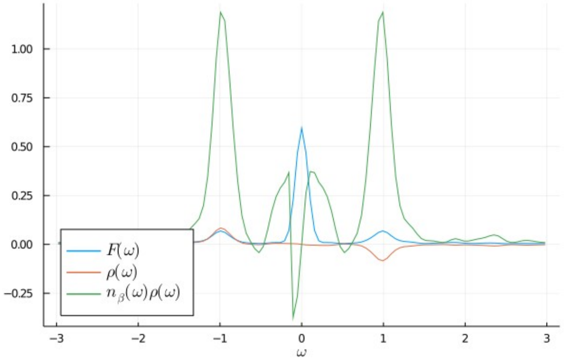
$$h = 0.5$$

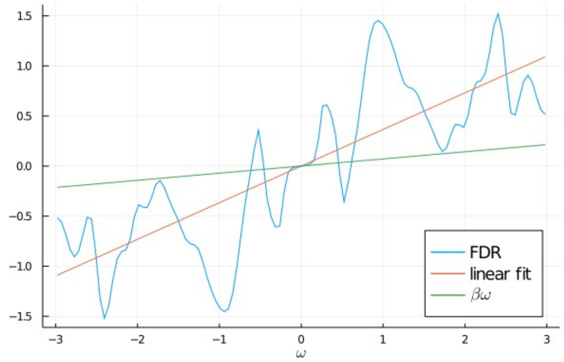
$$|\psi_0\rangle = |\uparrow\downarrow\ldots\uparrow\downarrow\rangle_x$$

$$\hat{A} = \hat{B} = \hat{S}_z^{(1)}$$

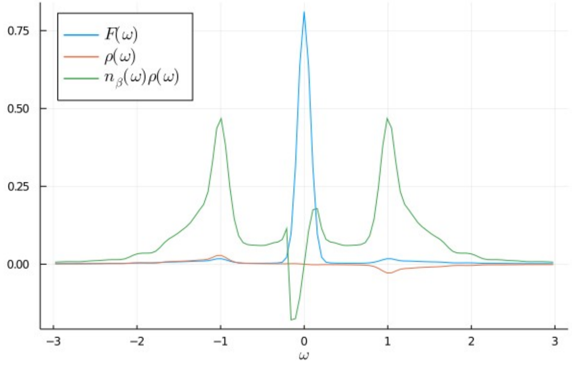
MBL Model (strong disorder – MBL phase)

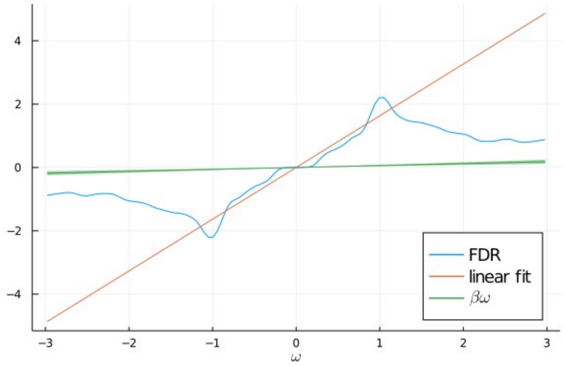
Single shot





Disorder average





$$F(\omega) = n_\beta(\omega)\rho(\omega)$$

Parameters

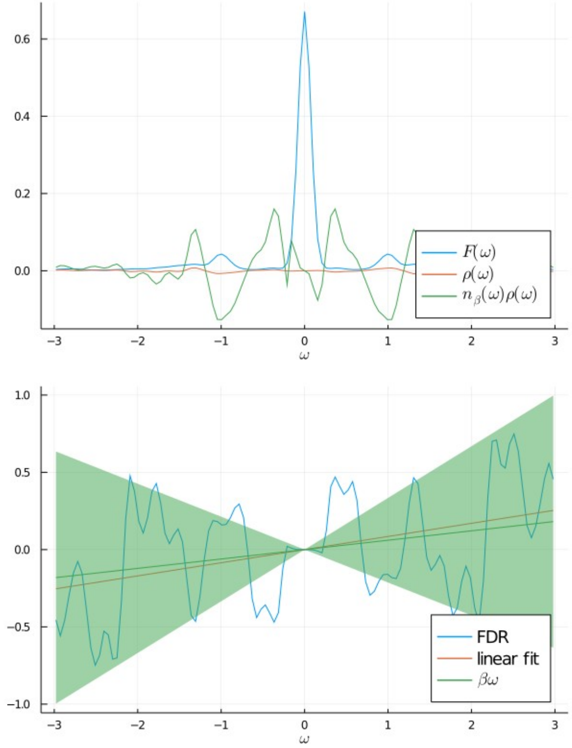
$$h = 6.5$$

$$|\psi_0\rangle = |\uparrow\downarrow\ldots\uparrow\downarrow\rangle_x$$

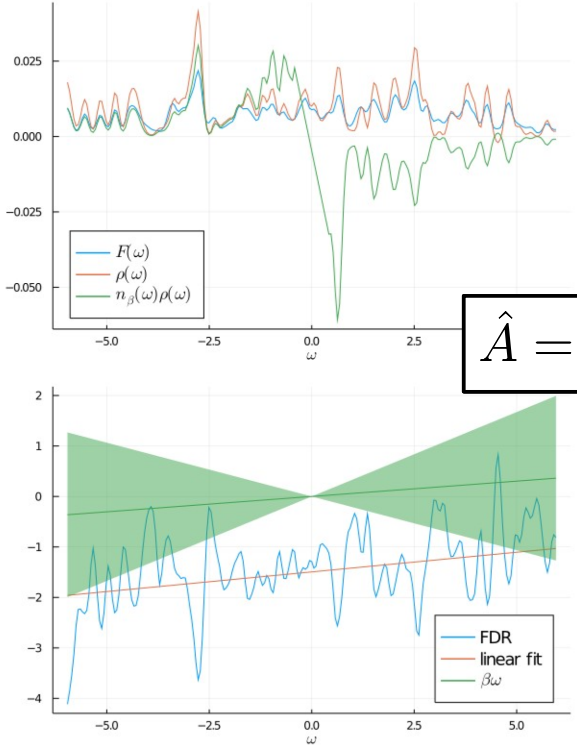
$$\hat{A} = \hat{B} = \hat{S}_z^{(1)}$$

MBL Model (strong disorder – MBL phase)

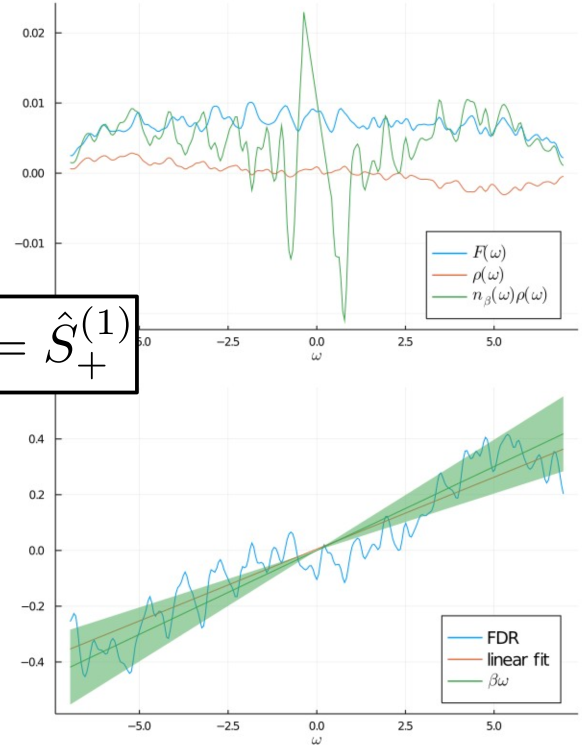
$$|\psi_0\rangle = |\uparrow\downarrow\ldots\uparrow\downarrow\rangle_z$$



$$|\psi_0\rangle = |\uparrow\downarrow\ldots\uparrow\downarrow\rangle_z$$



$$|\psi_0\rangle = |\uparrow\downarrow\ldots\uparrow\downarrow\rangle_x$$



$$\hat{A} = \hat{B}^\dagger = \hat{S}_+^{(1)}$$

Conclusion & Outlook

- Can use FDRs to show absence of thermalization
  - → What else can we learn?
- Rydberg-Experiment of Weidemüller group
  - → Glassy dynamics (failure of thermalization)
  - → MBL? Local conserved quantities?

$$\hat{H}_{XXZ} = \sum_{i,j} J_{i,j} \left(\hat{S}_x^{(i)} \hat{S}_x^{(j)} + \hat{S}_y^{(i)} \hat{S}_y^{(j)} + \delta \hat{S}_z^{(i)} \hat{S}_z^{(j)} \right)$$

$$J_{ij} \propto \frac{1}{|\vec{r}_i - \vec{r}_j|^{\alpha}}$$

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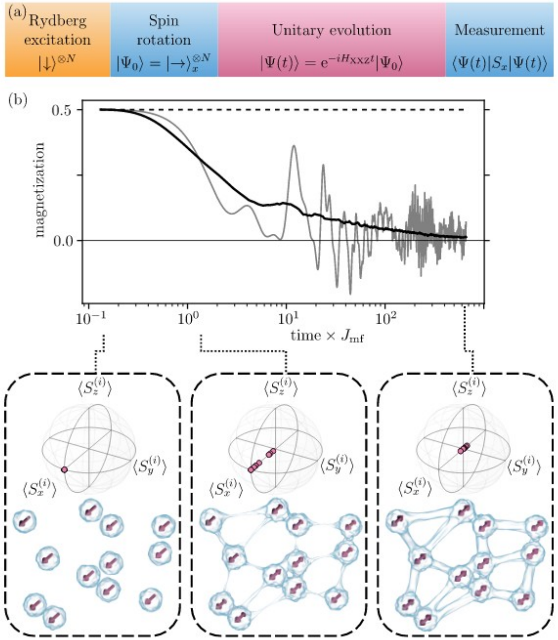

Signoles et al. Phys. Rev. X 11, 011011 (2021)